

# Effects of variable suction and thermophoresis on steady MHD combined free-forced convective heat and mass transfer flow over a semi-infinite permeable inclined plate in the presence of thermal radiation

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## Abstract

A two-dimensional steady MHD mixed convection and mass transfer flow over a semi-infinite porous inclined plate in the presence of thermal radiation with variable suction and thermophoresis has been analyzed numerically. The governing fundamental equations are approximated by a system of non-linear locally similar ordinary differential equations which are solved numerically by applying Nachtsheim–Swigert shooting iteration technique along with sixth-order Runge–Kutta integration scheme. Favorable comparison with previously published work is performed. Numerical results for the dimensionless velocity, temperature and concentration profiles as well as for the skin-friction coefficient, wall heat transfer and particle deposition rate are obtained and displayed graphically for pertinent parameters to show interesting aspects of the solutions. © 2007 Elsevier Masson SAS. All rights reserved.

**Keywords:** Mixed convection; Thermal radiation; MHD; Suction; Thermophoresis; Inclined plate

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## 1. Introduction

Radiative heat and mass transfer flow is very important in manufacturing industries for the design of reliable equipment, nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles. The effects of thermal radiation on the forced and free convection flows are also important in the context of space technology and processes involving high temperatures. In light of these various applications, England and Emery [1] studied the thermal radiation effect of an optically thin gray gas bounded by a stationary vertical plate. Raptis [2] studied radiation effect on the flow of a micropolar fluid past a continuously moving plate. Hossain and Takhar [3] analyzed the effect of radiation using the Rosseland diffusion approximation on mixed convection along a vertical plate with uniform free stream velocity and

surface temperature. Duwairi and Damseh [4,5], Duwairi [6], Damseh et al. [7] studied the effect of radiation and heat transfer in different geometry for various flow conditions. Recently Muthucumaraswamy et al. [8] studied the effect of thermal radiation on the unsteady free convective flow over a moving vertical plate with mass transfer in the presence of chemical reaction. The dimensionless governing equations were solved using the Laplace transform technique. Very recently Mbeledogu and Ogulu [9] studied heat and mass transfer on unsteady MHD natural convection flow of a rotating fluid past a vertical porous flat plate in the presence of radiative heat transfer. Badruddin et al. [10] studied the effect of viscous dissipation and radiation on natural convection in a porous medium embedded within vertical annulus.

However, the phenomenon of thermophoresis plays a vital role in the mass transfer mechanism of several devices involving small micron sized particles and large temperature gradients in the fields. Thermophoresis principle is utilized to manufacture graded index silicon dioxide and germanium dioxide optical fiber preforms used in the field of communications. Thermophoretic deposition of radioactive particles is considered to

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**Nomenclature**

$B_0$	magnetic induction	$T_\infty$	temperature of the ambient fluid . . . . . K
$C$	species concentration in the boundary layer . . . . . $\text{kg m}^{-2}$	$T_w$	temperature at the surface . . . . . K
$Cf_x$	local skin-friction coefficient	$U_\infty$	free stream velocity . . . . . $\text{m s}^{-1}$
$c_p$	specific heat due to constant pressure $\text{J kg}^{-1} \text{K}^{-1}$	$u, v$	the $x$ - and $y$ -component of the velocity field . . . . . $\text{m s}^{-1}$
$C_m, C_s, C_t$	constants in Eq. (10)	$v_w(x)$	transpiration velocity . . . . . $\text{m s}^{-1}$
$C_1, C_2, C_3$	constants in Eq. (10)	$V_T$	thermophoretic velocity . . . . . $\text{m s}^{-1}$
$C_\infty$	species concentration of the ambient fluid	$x, y$	axis in direction along and normal to the plate.. m
$D$	chemical molecular diffusivity	<i>Greek symbols</i>	
$f$	dimensionless stream function	$\alpha$	angle of inclination to vertical
$f_w$	dimensionless wall suction	$\beta$	volumetric coefficient of thermal expansion $1 \text{K}^{-1}$
$g$	acceleration due to gravity	$\gamma$	local buoyancy parameter
$Gr_x$	local Grashof number	$\rho$	fluid density . . . . . $\text{kg m}^{-3}$
$J_s$	rate of transfer of species concentration . . . . . $\text{kg m}^{-2} \text{s}^{-1}$	$\sigma$	electrical conductivity
$k$	thermophoretic coefficient defined by Eq. (10)	$\sigma_1$	Stefan–Boltzmann constant
$k_1$	mean absorption coefficient	$\psi$	stream function . . . . . $\text{m}^2 \text{s}^{-1}$
$Kn$	Knudsen number	$\eta$	similarity variable
$M$	magnetic field parameter	$\nu$	kinematic viscosity . . . . . $\text{m}^2 \text{s}^{-1}$
$Nu_x$	local Nusselt number	$\mu$	fluid viscosity . . . . . $\text{Pa s}$
$Pr$	Prandtl number	$\tau_w$	wall shear stress . . . . . Pa
$q_r$	radiative heat flux	$\theta$	dimensionless temperature
$q_w$	rate of heat transfer . . . . . W	$\phi$	dimensionless concentration
$Re_x$	local Reynolds number	$\lambda_g$	thermal conductivity of fluid
$Sc$	Schmidt number	$\lambda_p$	thermal conductivity of diffused particles
$St_x$	local Stanton number	$\tau$	thermophoretic parameter
$T$	temperature of the fluid in the boundary layer . . K		

be one of the important factors causing accidents in nuclear reactors. Goldsmith and May [11] first studied the thermophoretic transport involved in a simple one-dimensional flow for the measurement of the thermophoretic velocity. Thermophoresis in laminar flow over a horizontal flat plate has been studied theoretically by Goren [12]. Shen [13] analyzed the problem of thermophoretic deposition of small particles on to cold surfaces in two-dimensional and axi-symmetric cases. Thermophoresis in natural convection with variable properties for a laminar flow over a cold vertical flat plate has been studied by Jayaraj et al. [14]. Recently Selim et al. [15] studied the effect of surface mass flux on mixed convective flow past a heated vertical flat permeable plate with thermophoresis.

Therefore, the objective of the present paper is to investigate the effects of thermal radiation and thermophoresis on a two-dimensional steady MHD combined free-forced convective heat and mass transfer flow along a semi-infinite inclined permeable flat plate with variable suction.

## 2. Governing equations and similarity analysis

We consider a two-dimensional steady MHD laminar mixed convective flow of a viscous incompressible fluid along a semi-infinite inclined permeable flat plate with an acute angle  $\alpha$  to the vertical. With  $x$ -axis measured along the plate, a magnetic field of uniform strength  $B_0$  is applied in the  $y$ -direction which

is normal to the flow direction. Fluid suction is imposed at the plate surface. The external flow takes place in the direction parallel to the inclined plate and has the uniform velocity  $U_\infty$ . The temperature of the surface is held uniform at  $T_w$  which is higher than the ambient temperature  $T_\infty$ . The species concentration at the surface is maintained uniform at  $C_w$ , which is taken to be zero and that of the ambient fluid is assumed to be  $C_\infty$ . The effects of thermophoresis are being taken into account to help in the understanding of the mass deposition variation on the surface. The flow configuration and coordinate system are as shown in Fig. 1. We further assume that (i) the mass flux of particles is sufficiently small so that the main stream velocity and temperature fields are not affected by the thermo physical processes experienced by the relatively small number of particles, (ii) due to the boundary layer behavior the temperature gradient in the  $y$ -direction is much larger than that in the  $x$ -direction and hence only the thermophoretic velocity component which is normal to the surface is of importance, (iii) the fluid has constant kinematic viscosity and thermal diffusivity, and that the Boussinesq approximation may be adopted for steady laminar flow, (iv) the particle diffusivity is assumed to be constant, and the concentration of particles is sufficiently dilute to assume that particle coagulation in the boundary layer is negligible, (v) the magnetic Reynolds number is assumed to be small so that the induced magnetic field is negligible in

comparison to the applied magnetic field and (vi) the fluid is considered to be gray; absorbing-emitting radiation but non-scattering medium and the Rosseland approximation is used to describe the radiative heat flux in the  $x$ -direction is considered negligible in comparison to the  $y$ -direction.

Under the above assumptions, the governing equations (see Selim et al. [15]) for this problem can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (\text{Continuity}) \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) \cos \alpha - \frac{\sigma B_0^2}{\rho}(u - U_\infty) \quad (\text{Momentum}) \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\lambda_g}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} \quad (\text{Energy}) \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - \frac{\partial}{\partial y}(V_T C) \quad (\text{Diffusion}) \quad (4)$$

where  $u, v$  are the velocity components in the  $x$  and  $y$  directions respectively,  $\nu$  is the kinematic viscosity,  $g$  is the acceleration due to gravity,  $\sigma$  is the electrical conductivity,  $\rho$  is the density of the fluid,  $\beta$  is the volumetric coefficient of thermal expansion,  $T, T_w$  and  $T_\infty$  are the temperature of the fluid inside the thermal boundary layer, the plate temperature and the fluid temperature in the free stream, respectively, while  $C, C_w$  and  $C_\infty$  are the corresponding concentrations,  $B_0$  is the magnetic induction,  $\lambda_g$  is the thermal conductivity of fluid,  $c_p$  is the specific heat at constant pressure,  $q_r$  is the radiative heat flux in the  $y$ -direction,  $D$  is the molecular diffusivity of the species concentration and  $V_T$  is the thermophoretic velocity.

The appropriate boundary conditions for the above model are as follows:

$$u = 0, \quad v = v_w(x), \quad T = T_w, \quad C = C_w = 0 \quad \text{at } y = 0 \quad (5a)$$

$$u = U_\infty, \quad T = T_\infty, \quad C = C_\infty \quad \text{as } y \rightarrow \infty \quad (5b)$$

where  $U_\infty$  is the free stream velocity and  $v_w(x)$  represents the permeability of the porous surface.

The radiative heat flux  $q_r$  under Rosseland approximation has the form

$$q_r = -\frac{4\sigma_1}{3k_1} \frac{\partial T^4}{\partial y} \quad (6)$$

where  $\sigma_1$  is the Stefan–Boltzmann constant and  $k_1$  is the mean absorption coefficient.

We assume that the temperature difference within the flow are sufficiently small such that  $T^4$  may be expressed as a linear function of temperature. This is accomplished by expanding  $T^4$  in a Taylor series about  $T_\infty$  and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (7)$$

In the works [4–7] full radiation has been incorporated whereas the works ([2,8–10] and many others) used the linearized form (Eq. (7)) of thermal radiation. Linearized thermal radiation,

Eq. (7) makes the problem easy to handle. In this work we use linear form of thermal radiation.

Using (6) and (7) in Eq. (3) we have

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\lambda_g}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma_1 T_\infty^3}{3\rho c_p k_1} \frac{\partial^2 T}{\partial y^2} \quad (8)$$

Now the thermophoretic velocity  $V_T$ , which appears in Eq. (4), can be written as (see Talbot et al. [16]):

$$V_T = -k\nu \frac{\nabla T}{T_r} = -\frac{k\nu}{T_r} \frac{\partial T}{\partial y} \quad (9)$$

where  $T_r$  is some reference temperature and  $k$  is the thermophoretic coefficient which ranges in value from 0.2 to 1.2 as indicated by Batchelor and Shen [17] and is defined from the theory of Talbot et al. [16] by:

$$k = \frac{2C_s(\lambda_g/\lambda_p + C_t Kn)[1 + Kn(C_1 + C_2 e^{-C_3/Kn})]}{(1 + 3C_m Kn)(1 + 2\lambda_g/\lambda_p + 2C_t Kn)} \quad (10)$$

where  $C_1, C_2, C_3, C_m, C_s, C_t$  are constants,  $\lambda_g$  and  $\lambda_p$  are the thermal conductivities of the fluid and diffused particles, respectively and  $Kn$  is the Knudsen number.

In order to obtain similarity solution of the problem we introduce the following non-dimensional variables:

$$\eta = y \sqrt{\frac{U_\infty}{2\nu x}}, \quad \psi = \sqrt{2\nu x U_\infty} f(\eta) \\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C}{C_\infty} \quad (11a)$$

where  $\psi$  is the stream function that satisfies the continuity equation (1).

Since  $u = \frac{\partial \psi}{\partial y}$  and  $v = -\frac{\partial \psi}{\partial x}$  we have from Eq. (11a)

$$u = U_\infty f' \quad \text{and} \quad v = -\sqrt{\frac{\nu U_\infty}{2x}}(f - \eta f') \quad (11b)$$

Here prime denotes ordinary differentiation with respect to the similarity variable  $\eta$ .

Now substituting Eq. (11) in Eqs. (2)–(4) we obtain the following ordinary differential equations which are locally similar:

$$f''' + ff'' + \gamma \theta \cos \alpha - M(f' - 1) = 0 \quad (12)$$

$$(3R + 4)\theta'' + 3RPr f\theta' = 0 \quad (13)$$

$$\phi'' + Sc(f - \tau\theta')\phi' - Sc\tau\phi\theta'' = 0 \quad (14)$$

The boundary conditions (5) then turn into

$$f = f_w, \quad f' = 0, \quad \theta = 1, \quad \phi = 0 \quad \text{at } \eta = 0 \quad (15a)$$

$$f' = 1, \quad \theta = 0, \quad \phi = 1 \quad \text{as } \eta \rightarrow \infty \quad (15b)$$

where  $f_w = -v_w(x) \sqrt{\frac{2x}{\nu U_\infty}}$  is the dimensionless wall suction velocity at the permeable plate.

The dimensionless parameters introduced in the above equations are defined as follows:

$\gamma = \frac{Gr_x}{Re_x^2}$  is the local buoyancy parameter,  $M = \frac{\sigma B_0^2 2x}{\rho U_\infty}$  is the local Magnetic field parameter,  $Gr_x = \frac{g\beta(T_w - T_\infty)(2x)^3}{\nu^2}$  is the local Grashof number,  $Re_x = \frac{U_\infty 2x}{\nu}$  is the local Reynolds number,

$Pr = \frac{\nu \rho c_p}{\lambda_g}$  is the Prandtl number,  $R = \frac{\lambda_g k_1}{4\sigma_1 T_\infty^3}$  is the radiation parameter,  $Sc = \frac{\nu}{D}$  is the Schmidt number and  $\tau = -\frac{k(T_w - T_\infty)}{T_r}$  is the thermophoretic parameter.

The skin-friction coefficient, wall heat transfer coefficient (or local Nusselt number) and wall deposition flux (or the local Stanton number) are important physical parameters. These can be obtained from the following expressions:

$$Cf_x Re_x^{1/2} = \frac{\tau_w}{\rho U_\infty^2} = f''(0); \quad \tau_w = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0} \quad (16)$$

$$Nu_x Re_x^{-1/2} = \frac{xq_w}{(T_w - T_\infty)\lambda_g} = -\frac{1}{2} \left[ 1 + \frac{4}{3R} \right] \theta'(0);$$

$$q_w = -\lambda_g \left( \frac{\partial T}{\partial y} \right)_{y=0} - \frac{4\sigma_1}{3k_1} \left( \frac{\partial T^4}{\partial y} \right)_{y=0} \quad (17)$$

$$St_x Sc Re_x^{1/2} = -\frac{J_s}{U_\infty C_\infty} = \phi'(0);$$

$$J_s = -D \left( \frac{\partial C}{\partial y} \right)_{y=0} \quad (18)$$

### 3. Numerical solution

The set of non-linear and locally similar ordinary differential equations (12)–(14) with boundary conditions (15) have been solved numerically by using sixth order Runge–Kutta method along with Nachtsheim–Swigert [18] shooting iteration technique (for detailed discussion of the method see Maleque and Sattar [19] and Alam et al. [20]) with  $\gamma$ ,  $M$ ,  $Pr$ ,  $Sc$ ,  $R$ ,  $f_w$ ,  $\tau$  and  $\alpha$  as prescribed parameters. A step size of  $\Delta\eta = 0.01$  was selected to be satisfactory for a convergence criterion of  $10^{-6}$  in all cases. The value of  $\eta_\infty$  was found to each iteration loop by the statement  $\eta_\infty = \eta_\infty + \Delta\eta$ . The maximum value of  $\eta_\infty$  to each group of parameters  $\gamma$ ,  $M$ ,  $Pr$ ,  $Sc$ ,  $R$ ,  $f_w$ ,  $\tau$  and  $\alpha$  is determined when the value of the unknown boundary conditions at  $\eta = 0$  not change to successful loop with error less than  $10^{-6}$ .

To assess the accuracy of our code, the present result has been compared with White [21], when  $M = f_w = \gamma = R = 0$  and  $\alpha = 90^\circ$  (see Fig. 2). From this figure we see an excellent agreement between them.

In order to see the effect of step size ( $\Delta\eta$ ) we ran the code for our model with three different step sizes as  $\Delta\eta = 0.01$ ,  $\Delta\eta = 0.004$ ,  $\Delta\eta = 0.001$  and in each case we found very good agreement among them. Fig. 3 shows the velocity profiles for different step sizes.

### 4. Results and discussion

Numerical calculations have been carried out for different values of  $\gamma$ ,  $f_w$ ,  $R$ ,  $Sc$ ,  $\tau$ ,  $Pr$ ,  $M$  and  $\alpha$ . The case  $\gamma \gg 1$  corresponds to pure free convection,  $\gamma = 1$  corresponds to mixed (combined free-forced) convection and  $\gamma \ll 1$  corresponds to pure forced convection. Throughout this calculation we have considered  $\gamma = 1.0$  unless otherwise specified.

Figs. 4(a)–(c) illustrate the influence of the suction parameter  $f_w$  on the velocity, temperature and concentration profiles,

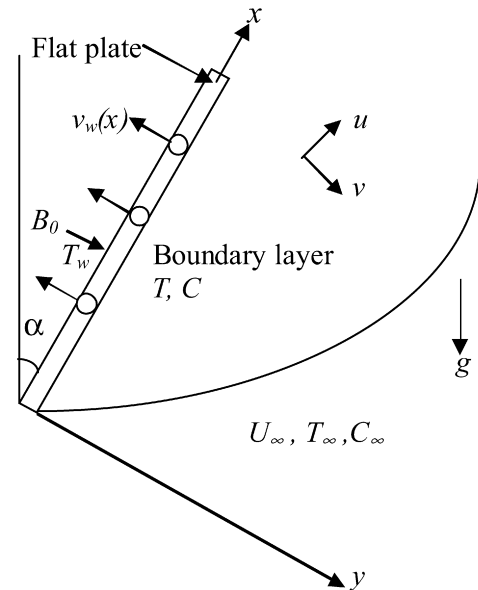


Fig. 1. Flow configuration and coordinate system.

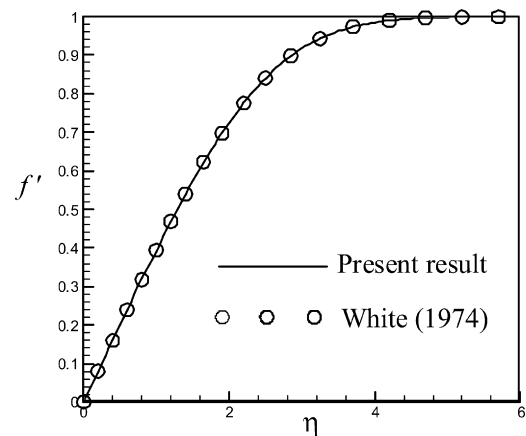


Fig. 2. Comparison of velocity profiles with White [21].

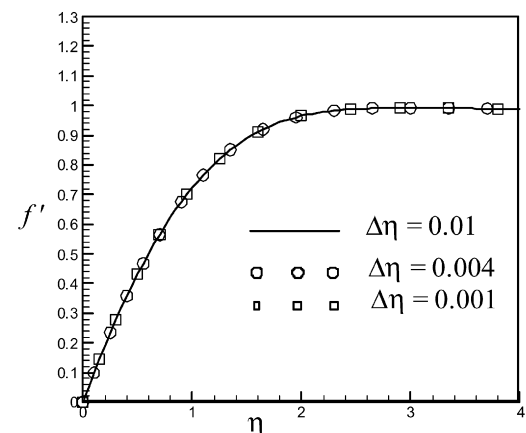


Fig. 3. Velocity profiles for different values of  $\Delta\eta$ .

respectively. The imposition of wall fluid suction for this problem has the effect of reducing all the hydrodynamic, thermal and concentration boundary layers causing the fluid velocity

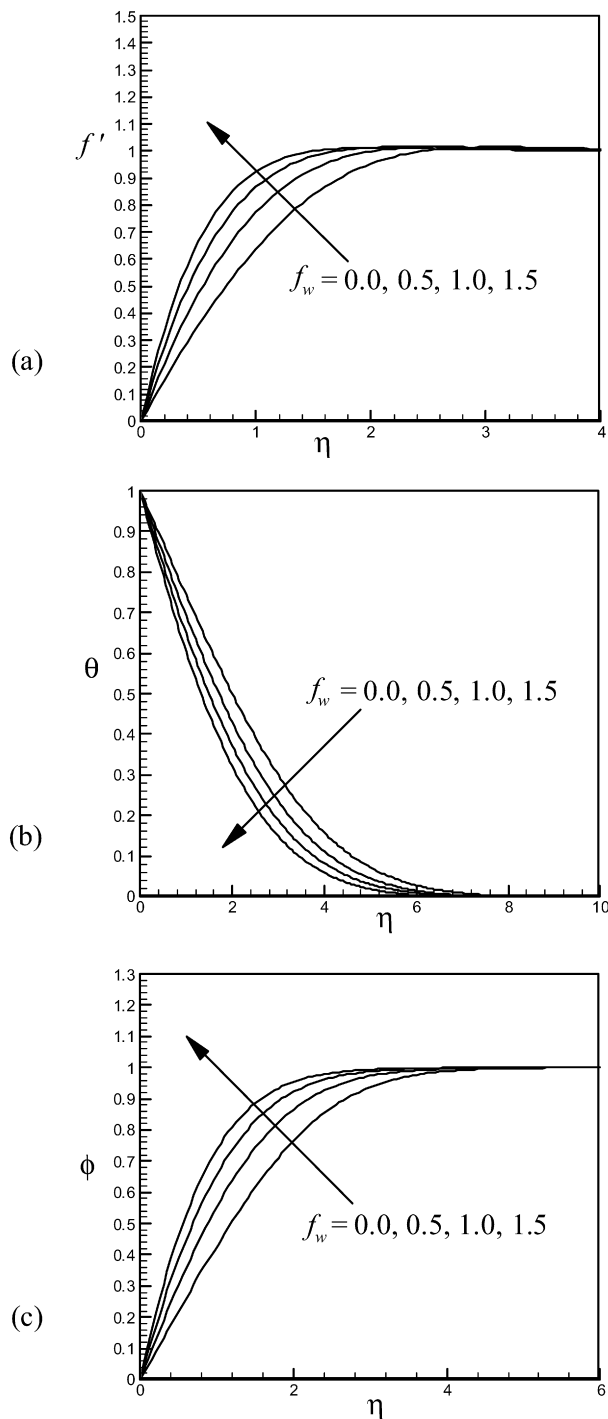


Fig. 4. (a)–(c) Dimensionless velocity, temperature and concentration profiles, respectively for different values of  $f_w$  and for  $\gamma = 1.0$ ,  $Pr = 0.70$ ,  $Sc = 0.60$ ,  $R = 0.40$ ,  $M = 0.20$ ,  $\tau = 0.1$  and  $\alpha = 30^\circ$ .

and its concentration to increase while decreasing its temperature. The decreasing thickness of the concentration layer is caused by two effects; (i) the direct action of suction, and (ii) the indirect action of suction causing a thinner thermal boundary layer, which corresponds to higher temperature gradient, a consequent increase in the thermophoretic force and higher concentration gradient.

Figs. 5(a)–(c) show the dimensionless velocity, temperature and concentration profiles for different values of radiation para-

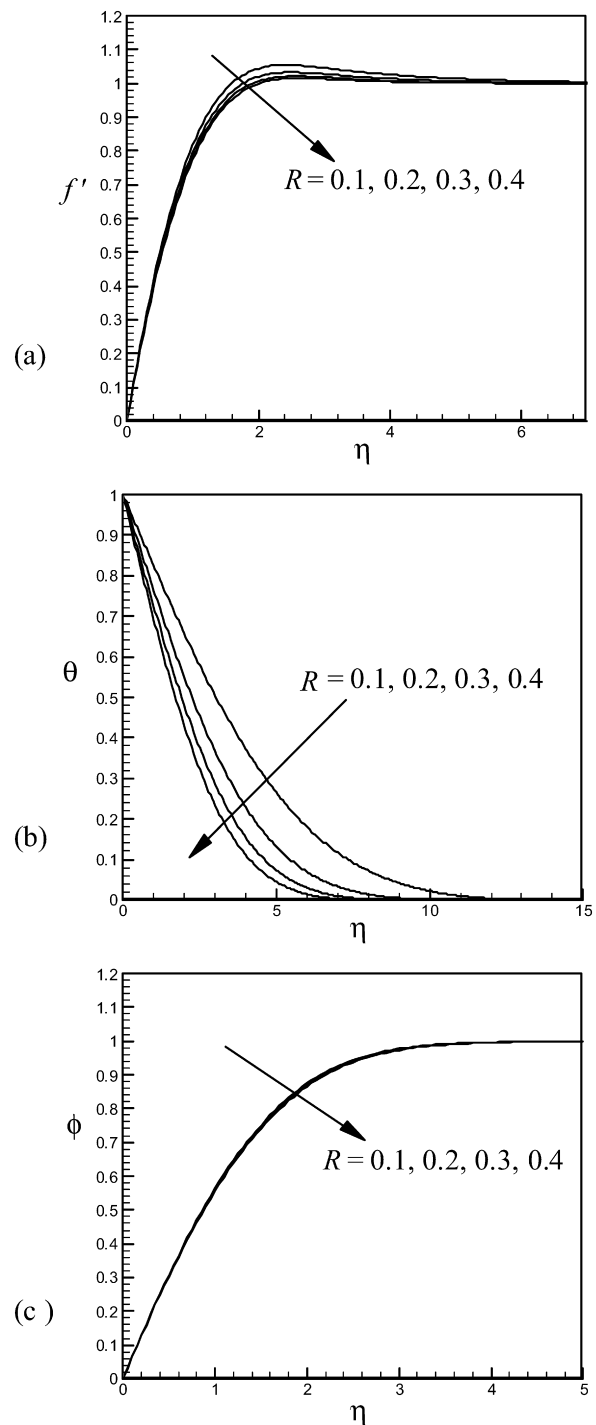


Fig. 5. (a)–(c) Dimensionless velocity, temperature and concentration profiles, respectively for different values of  $R$  and for  $\gamma = 1.0$ ,  $Pr = 0.70$ ,  $Sc = 0.60$ ,  $f_w = 0.50$ ,  $M = 0.20$ ,  $\tau = 0.1$  and  $\alpha = 30^\circ$ .

meter  $R$  for mixed convection region respectively. From these figures we see that velocity, temperature and concentration decrease as the radiation parameter  $R$  increases.

Fig. 6 shows typical concentration profiles for various values of the Schmidt number  $Sc$ . It is clear from this figure that the concentration boundary layer thickness decreases as the Schmidt number  $Sc$  increases and this is the analogous to the effect of increasing the Prandtl number on the thickness of a thermal boundary layer.

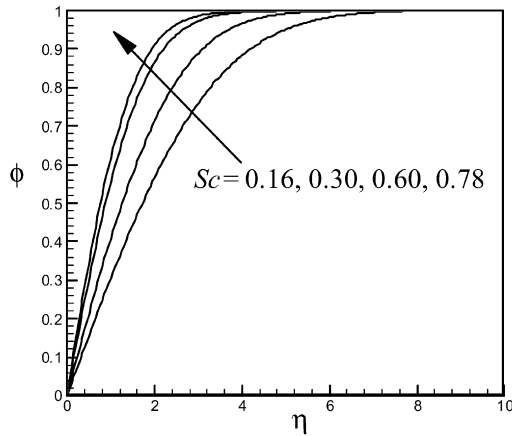


Fig. 6. Dimensionless concentration profiles for different values of  $Sc$  and for  $\gamma = 1.0$ ,  $Pr = 0.70$ ,  $R = 0.40$ ,  $f_w = 0.50$ ,  $M = 0.20$ ,  $\tau = 0.1$  and  $\alpha = 30^\circ$ .

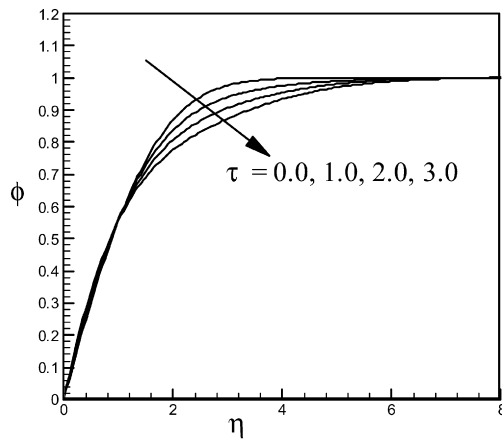


Fig. 7. Dimensionless concentration profiles for different values of  $\tau$  and for  $\gamma = 1.0$ ,  $Pr = 0.70$ ,  $Sc = 0.60$ ,  $f_w = 0.50$ ,  $M = 0.20$ ,  $R = 0.4$  and  $\alpha = 30^\circ$ .

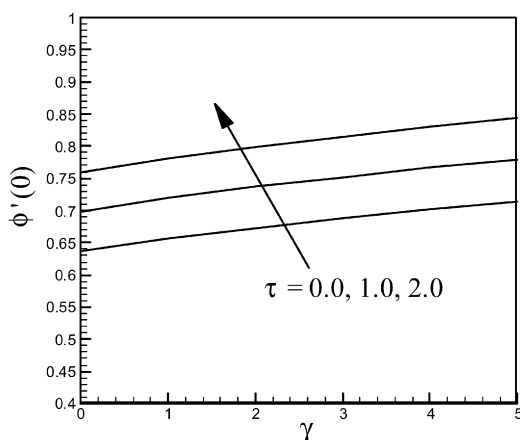


Fig. 8. Effects of  $\tau$  and  $\gamma$  on wall deposition flux for  $Pr = 0.70$ ,  $Sc = 0.60$ ,  $M = 0.20$ ,  $R = 0.4$ ,  $f_w = 0.50$  and  $\alpha = 30^\circ$ .

The effect of thermophoretic parameter  $\tau$  on the concentration field is shown in Fig. 7. For the parametric conditions used in Fig. 7, the effect of increasing the thermophoretic parameter  $\tau$  is limited to increasing slightly the wall slope of the concentration profiles for  $\eta < 1.2$  but decreasing the concentration for

values of  $\eta > 1.2$ . This is true only for small values of  $Sc$  for which the Brownian diffusion effect is large compared to the convection effect. However, for large values of  $Sc$  ( $Sc > 1000$ ) the diffusion effect is minimal compared to the convection effect and, therefore, the thermophoretic parameter  $\tau$  is expected to alter the concentration boundary layer significantly. This is consistent with the work of Goren [12] on thermophoresis of aerosol particles in flat plate boundary layer.

In Fig. 8 we have shown the combined effect of thermophoretic parameter  $\tau$  and buoyancy parameter  $\gamma$  on the wall deposition flux  $St_x Sc Re_x^{1/2}$ . From this figure we observe that for all values of  $\gamma$ , the surface mass flux increases as the thermophoretic parameter  $\tau$  increases.

Figs. 9(a)–(c) reveal the effects of suction parameter  $f_w$  and buoyancy parameter  $\gamma$  on the local skin-friction coefficient, wall heat transfer and wall deposition flux, respectively. These figures confirm that as  $f_w$  increases, the local skin-friction coefficient, the wall heat transfer and the wall deposition flux all increase.

Figs. 10(a)–(c), respectively, show the effects of Prandtl number  $Pr$  and radiation parameter  $R$  on the local skin-friction coefficient, rate of heat transfer and wall deposition flux. From these figures we observe that both the local skin-friction coefficient and wall deposition flux decreases with the increase of Prandtl number  $Pr$  whereas heat transfer rate increases with the increase of both the Prandtl number  $Pr$  and radiation parameter  $R$ . For large Prandtl number flows radiation has significant increasing effect on the rate of heat transfer.

The effects of the magnetic field parameter  $M$  and angle of inclination  $\alpha$  on the local skin-friction coefficient, wall heat transfer and wall deposition flux are shown in Figs. 11(a)–(c) respectively. From Fig. 11(a) we see that the local skin-friction coefficient is found to increase due to an increase in the magnetic field strength when the angle of inclination is fixed. This is expected, since the application of a magnetic field moving with the free stream has the tendency to induce a motive force which increases the motion of the fluid and hence increases the surface friction force. From this figure we also observe that an increase in the angle of inclination would produce a decrease in the buoyancy force and hence reduce the local skin-friction coefficient for all values of  $M$ . From Figs. 11 (b), (c) we see that both the wall heat transfer as well as the wall deposition flux decreases with the increasing values of the angle of inclination whereas these coefficients increase with an increasing values of the magnetic field parameter.

## 5. Conclusions

In this paper we have studied numerically the effects of variable suction and thermal radiation on a steady MHD mixed convective flows of a viscous incompressible fluid along a semi-infinite permeable inclined plate in the presence of thermophoresis. A set of similarity equations governing the fluid velocity, temperature and the particle mass concentration was obtained by using similarity transformations. The resulting nonlinear and locally similar ordinary differential equations have been solved numerically by applying shooting method. Com-

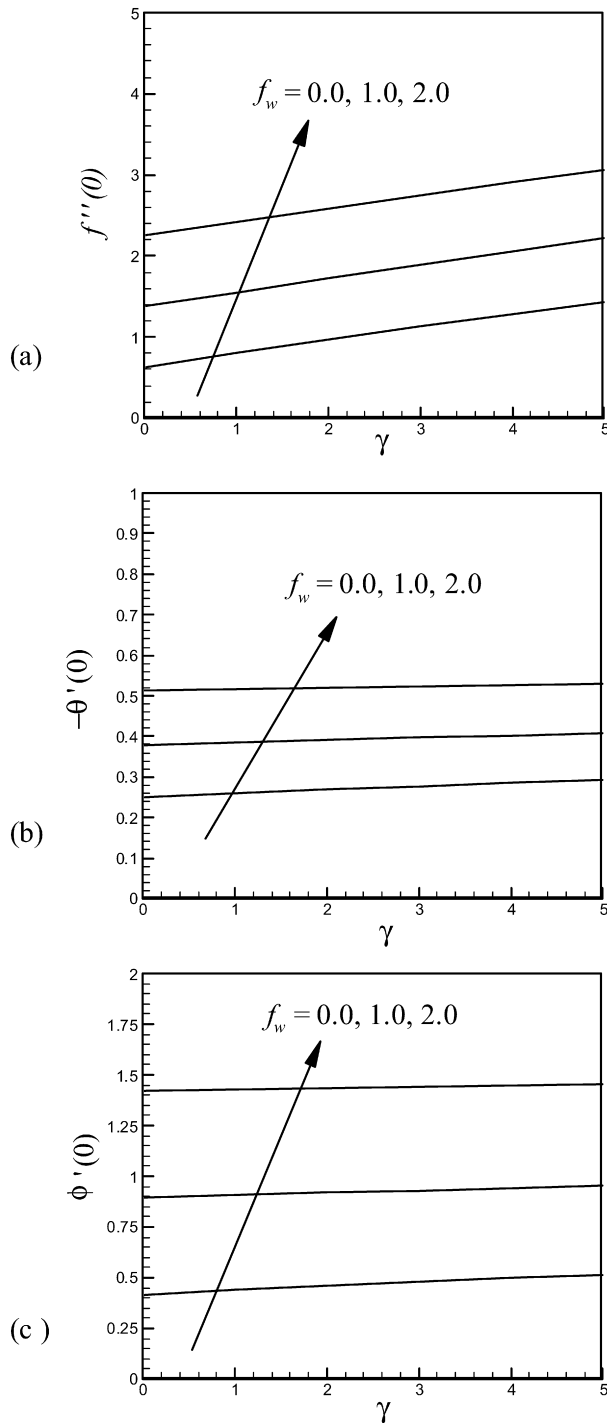


Fig. 9. (a)–(c) Effects of  $f_w$  and  $\gamma$  on skin-friction coefficient, wall heat transfer and wall deposition flux, respectively, for  $Pr = 0.70$ ,  $Sc = 0.60$ ,  $M = 0.20$ ,  $R = 0.4$ ,  $\tau = 0.10$  and  $\alpha = 30^\circ$ .

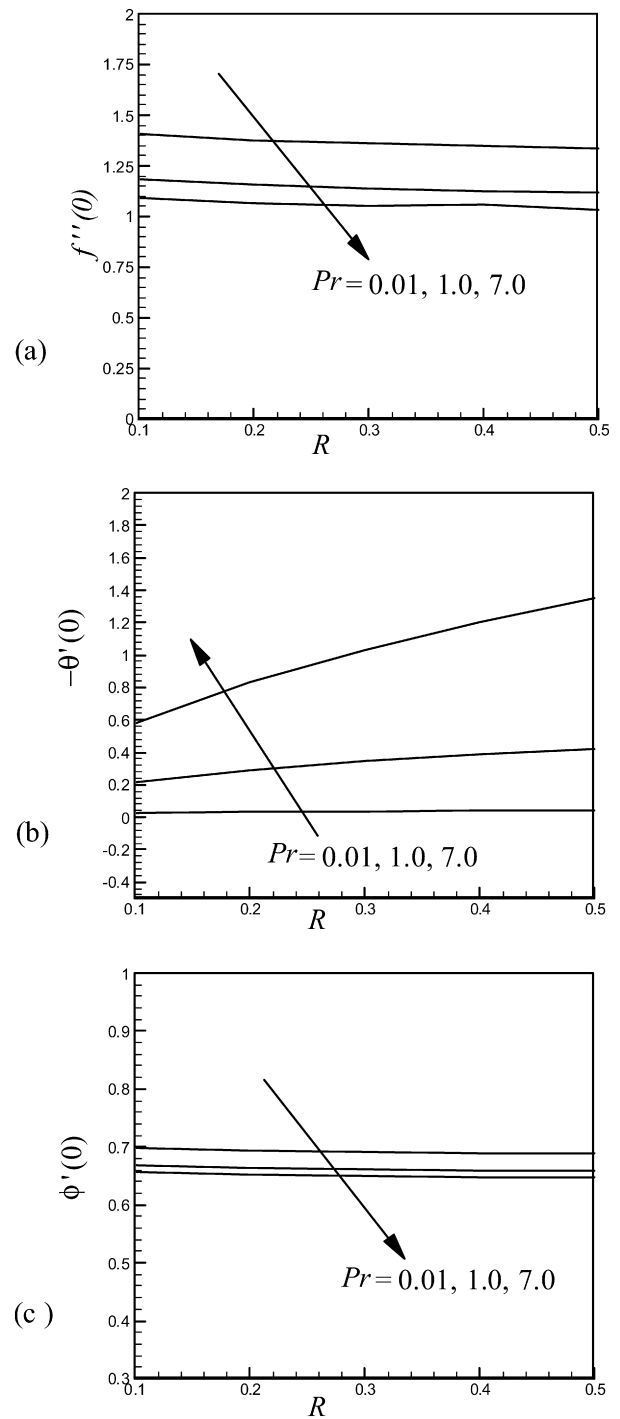


Fig. 10. (a)–(c) Effects of  $Pr$  and  $R$  on skin-friction coefficient, wall heat transfer and wall deposition flux, respectively, for  $\gamma = 1.0$ ,  $Pr = 0.70$ ,  $Sc = 0.60$ ,  $M = 0.20$ ,  $f_w = 0.5$ ,  $\tau = 0.10$  and  $\alpha = 30^\circ$ .

parison with previously published work was performed and the results were found to be in excellent agreement. From the present numerical investigation the following conclusions can be drawn:

- i. In mixed convection regime all the hydrodynamic, thermal and concentration boundary layers decrease as the suction parameter increases.
- ii. In mixed convection regime fluid velocity and concentration boundary layer increases whereas thermal boundary layer decreases with the increase of the radiation parameter.
- iii. As the Schmidt number increases, the concentration boundary layer decreases.
- iv. In mixed convection regime the surface mass flux increases with the increase of the thermophoretic parameter.

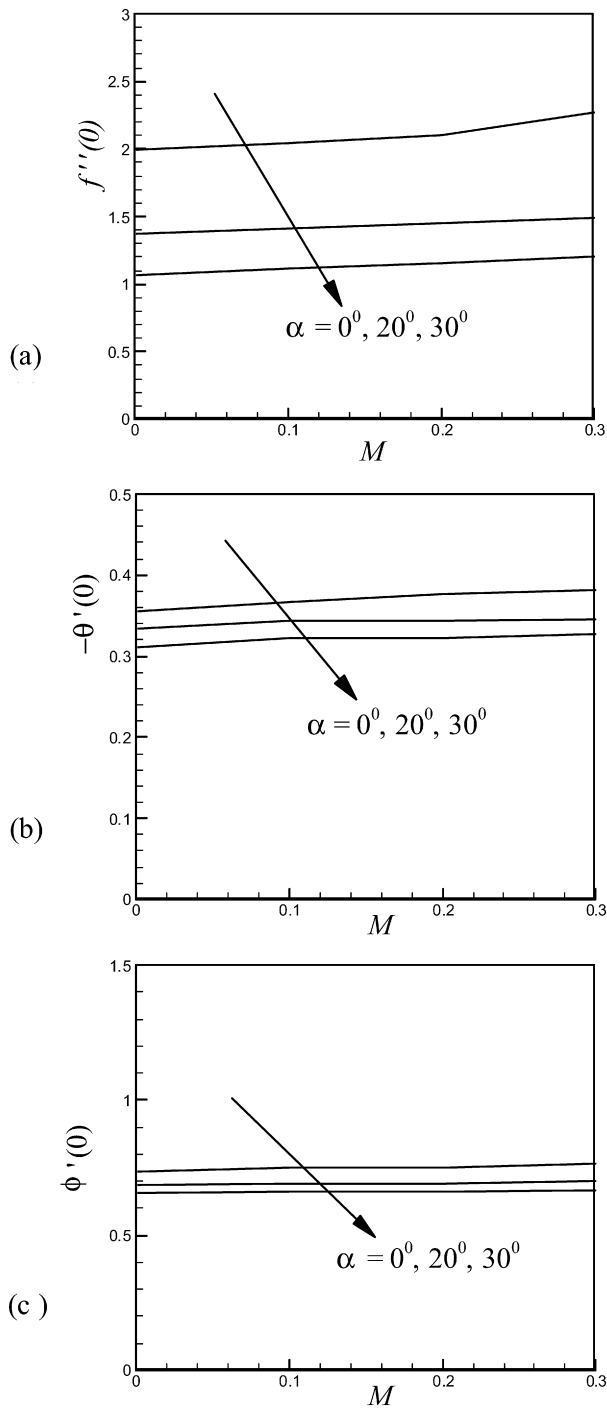


Fig. 11. (a)–(c) Effects of  $\alpha$  and  $M$  on skin-friction coefficient, wall heat transfer and wall deposition flux, respectively, for  $\gamma = 1.0$ ,  $Pr = 0.70$ ,  $Sc = 0.60$ ,  $f_w = 0.5$ ,  $\tau = 0.10$  and  $R = 0.40$ .

- v. Increasing the angle of inclination has the effect to decrease the local skin-friction coefficient, the local Nusselt number and the local Stanton number.

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